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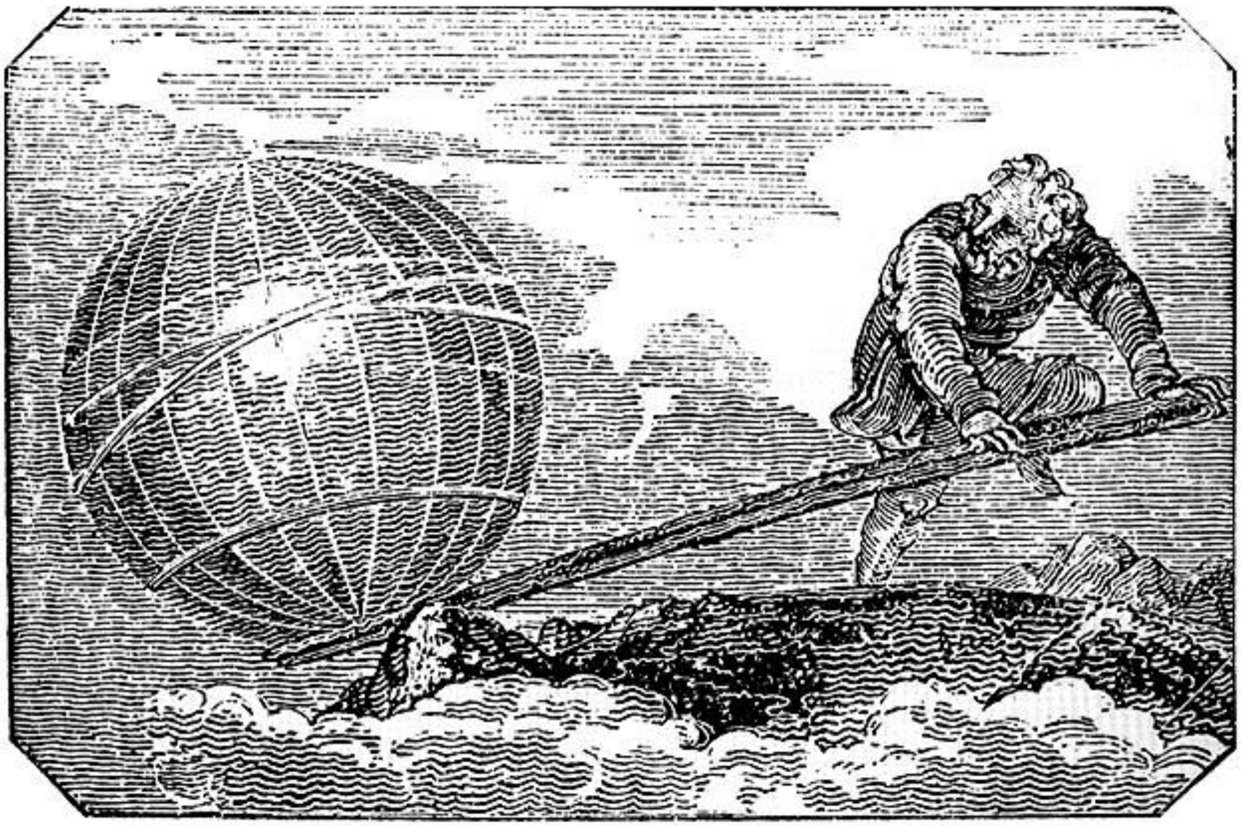
樹莓派, 樹莓派之學習, 樹莓派之教育

# STEM 隨筆︰古典力學︰運動學【九】

2018-08-21 | 懸鉤子 | 發表迴響



1824年，在倫敦發行的《機械雜誌》內的一副刻畫。阿基米德說：「給我一個支點，我就可以撬起整個地球。」

手無縛雞之力的人，能舉重乎？

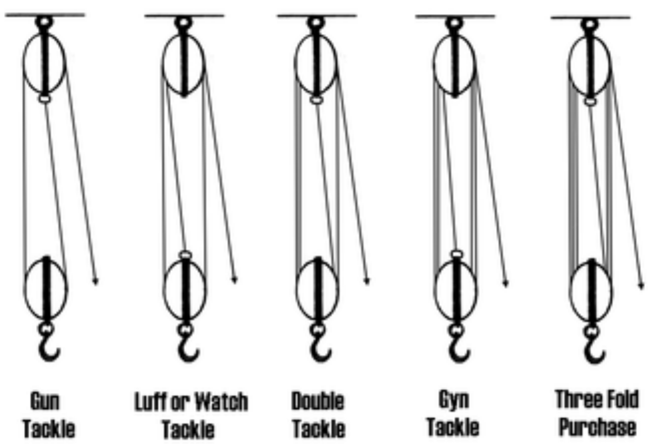
倘其使用『滑輪』，易如反掌呦！

此乃『工具』設計者之『目的』也☆

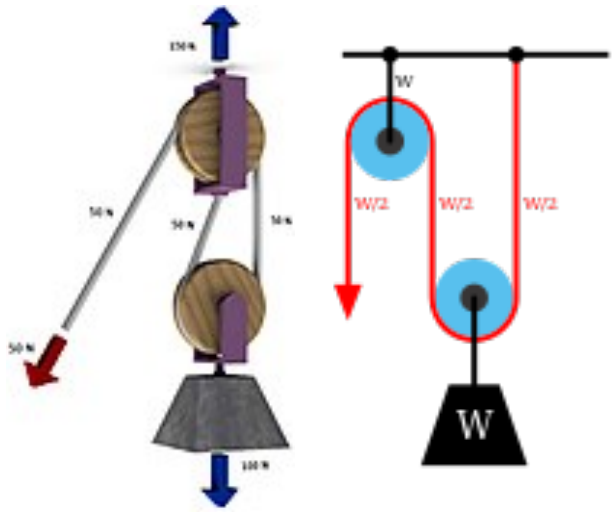
## Block and tackle

A block and tackle is an assembly of a rope and pulleys that is used to lift loads. A number of pulleys are assembled together to form the blocks, one that is fixed and one that moves with the load. The rope is threaded through the pulleys to provide mechanical advantage that amplifies that force applied to the rope.[4]

In order to determine the mechanical advantage of a block and tackle system consider the simple case of a gun tackle, which has a single mounted, or fixed, pulley and a single movable pulley. The rope is threaded around the fixed block and falls down to the moving block where it is threaded around the pulley and brought back up to be knotted to the fixed block.




The mechanical advantage of a block and tackle equals the number of sections of rope that support the moving block; shown here it is 2, 3, 4, 5, and 6, respectively.




Let *S* be the distance from the axle of the fixed block to the end of the rope, which is *A* where the input force is applied. Let *R* be the distance from the axle of the fixed block to the axle of the moving block, which is *B* where the load is applied.

The total length of the rope *L* can be written as

$$L = 2R + S + K,$$

where *K* is the constant length of rope that passes over the pulleys and does not change as the block and tackle moves.

The velocities $V_A$ and $V_B$ of the points *A* and *B* are related by the constant length of the rope, that is

$$\dot{L} = 2\dot{R} + \dot{S} = 0,$$

or

$$\dot{S} = -2\dot{R}.$$

The negative sign shows that the velocity of the load is opposite to the velocity of the applied force, which means as we pull down on the rope the load moves up.

Let $V_A$ be positive downwards and $V_B$ be positive upwards, so this relationship can be written as the speed ratio

$$\frac{V_A}{V_B} = \frac{\dot{S}}{-\dot{R}} = 2,$$

where 2 is the number of rope sections supporting the moving block.

Let $F_A$ be the input force applied at *A* the end of the rope, and let $F_B$ be the force at *B* on the moving block. Like the velocities $F_A$ is directed downwards and $F_B$ is directed upwards.

For an ideal block and tackle system there is no friction in the pulleys and no deflection or wear in the rope, which means the power input by the applied force $F_A V_A$ must equal the power out acting on the load $F_B V_B$, that is

$$F_A V_A = F_B V_B.$$

The ratio of the output force to the input force is the mechanical advantage of an ideal gun tackle system,

$$MA = \frac{F_B}{F_A} = \frac{V_A}{V_B} = 2.$$

This analysis generalizes to an ideal block and tackle with a moving block supported by *n* rope sections,

$$MA = \frac{F_B}{F_A} = \frac{V_A}{V_B} = n.$$

This shows that the force exerted by an ideal block and tackle is *n* times the input force, where *n* is the number of sections of rope that support the moving block.

所以阿基米德豪情壯志，想移動地球哩！？

## Lever

A **lever** (/ˈliːvər/ or US: /ˈlɛvər/) is a simple machine consisting of a beam or rigid rod pivoted at a fixed hinge, or **fulcrum**. A lever is a rigid body capable of rotating on a point on itself. On the basis of the location of fulcrum, load and effort, the lever is divided into three types. It is one of the six simple machines identified by Renaissance scientists. A lever amplifies an input force to provide a greater output force, which is said to provide**leverage**. The ratio of the output force to the input force is the mechanical advantage of the lever.

## Etymology

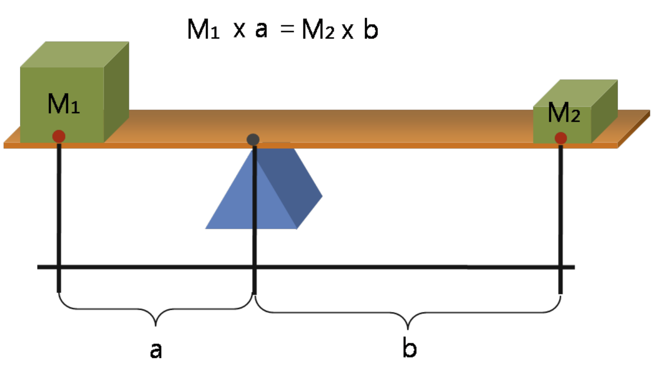
The word "lever" entered English about 1300 from Old French, in which the word was *levier*. This sprang from the stem of the verb *lever*, meaning "to raise". The verb, in turn, goes back to the Latin *levare*, itself from the adjective *levis*, meaning "light" (as in "not heavy"). The word's primary origin is the Proto-Indo-European (PIE) stem *legwh-*, meaning "light", "easy" or "nimble", among other things. The PIE stem also gave rise to the English word "light".[1]

## Early use

The earliest remaining writings regarding levers date from the 3rd century BCE and were provided by Archimedes. 'Give me a place to stand, and I shall move the Earth with it' is a remark of Archimedes who formally stated the correct mathematical principle of levers (quoted by Pappus of Alexandria).

It is assumed[*by whom?*] that in ancient Egypt, constructors used the lever to move and uplift obelisks weighing more than 100 tons.

## Force and levers




A lever in balance

A lever is a beam connected to ground by a hinge, or pivot, called a fulcrum. The ideal lever does not dissipate or store energy, which means there is no friction in the hinge or bending in the beam. In this case, the power into the lever equals the power out, and the ratio of output to input force is given by the ratio of the distances from the fulcrum to the points of application of these forces. This is known as the *law of the lever.*[citation needed]

The mechanical advantage of a lever can be determined by considering the balance of moments or torque, *T*, about the fulcrum.

$$T_2 = F_2 b$$

where $F_1$ is the input force to the lever and $F_2$ is the output force. The distances *a* and *b* are the perpendicular distances between the forces and the fulcrum.

Since the moments of torque must be balanced, $T_1 = T_2$. So, $F_1 a = F_2 b$.

The mechanical advantage of the lever is the ratio of output force to input force,

$$MA = \frac{F_2}{F_1} = \frac{a}{b}.$$

This relationship shows that the mechanical advantage can be computed from ratio of the distances from the fulcrum to where the input and output forces are applied to the lever, assuming no losses due to friction, flexibility or wear. This remains true even though the*horizontal* distance (perpendicular to the pull of gravity) of both *a* and *b* change (diminish) as the lever changes to any position away from the horizontal.

若以今日『物理概念』思之，實需了解雖說『能量守恆』，那個『能量』的『利用方式』為何不可以不同勒？！

## Virtual work and the law of the lever

A lever is modeled as a rigid bar connected to a ground frame by a hinged joint called a fulcrum. The lever is operated by applying an input force $\mathbf{F}_A$ at a point *A* located by the coordinate vector $\mathbf{r}_A$ on the bar. The lever then exerts an output force $\mathbf{F}_B$ at the point *B*located by $\mathbf{r}_B$. The rotation of the lever about the fulcrum $P$ is defined by the rotation angle $\theta$ in radians.

Let the coordinate vector of the point $P$ that defines the fulcrum be $\mathbf{r}_P$, and introduce the lengths

$$a = |\mathbf{r}_A - \mathbf{r}_P|, \quad b = |\mathbf{r}_B - \mathbf{r}_P|,$$

which are the distances from the fulcrum to the input point $A$ and to the output point $B$, respectively.

Now introduce the unit vectors $\mathbf{e}_A$ and $\mathbf{e}_B$ from the fulcrum to the point $A$ and $B$, so

$$\mathbf{r}_A - \mathbf{r}_P = a\mathbf{e}_A, \quad \mathbf{r}_B - \mathbf{r}_P = b\mathbf{e}_B.$$

The velocity of the points $A$ and $B$ are obtained as

$$\mathbf{v}_A = \dot{\theta} a \mathbf{e}_A^\perp, \quad \mathbf{v}_B = \dot{\theta} b \mathbf{e}_B^\perp,$$

where $\mathbf{e}_A{}^\perp$ and $\mathbf{e}_B{}^\perp$ are unit vectors perpendicular to $\mathbf{e}_A$ and $\mathbf{e}_B$, respectively.

The angle $\theta$ is the generalized coordinate that defines the configuration of the lever, and the generalized force associated with this coordinate is given by

$$F_\theta = \mathbf{F}_A \cdot \frac{\partial \mathbf{v}_A}{\partial \dot{\theta}} - \mathbf{F}_B \cdot \frac{\partial \mathbf{v}_B}{\partial \dot{\theta}} = a(\mathbf{F}_A \cdot \mathbf{e}_A^\perp) - b(\mathbf{F}_B \cdot \mathbf{e}_B^\perp) = aF_A - bF_B,$$

where $F_A$ and $F_B$ are components of the forces that are perpendicular to the radial segments $PA$ and $PB$. The principle of virtual work states that at equilibrium the generalized force is zero, that is

$$F_\theta = aF_A - bF_B = 0.$$

Thus, the ratio of the output force $F_B$ to the input force $F_A$ is obtained as

$$MA = \frac{F_B}{F_A} = \frac{a}{b},$$

which is the mechanical advantage of the lever.

This equation shows that if the distance *a* from the fulcrum to the point *A* where the input force is applied is greater than the distance *b* from fulcrum to the point *B* where the output force is applied, then the lever amplifies the input force. If the opposite is true that the distance from the fulcrum to the input point *A* is less than from the fulcrum to the output point *B*, then the lever reduces the magnitude of the input force.

故而知『凡有所得，亦有所失』夫？？

# Gear train

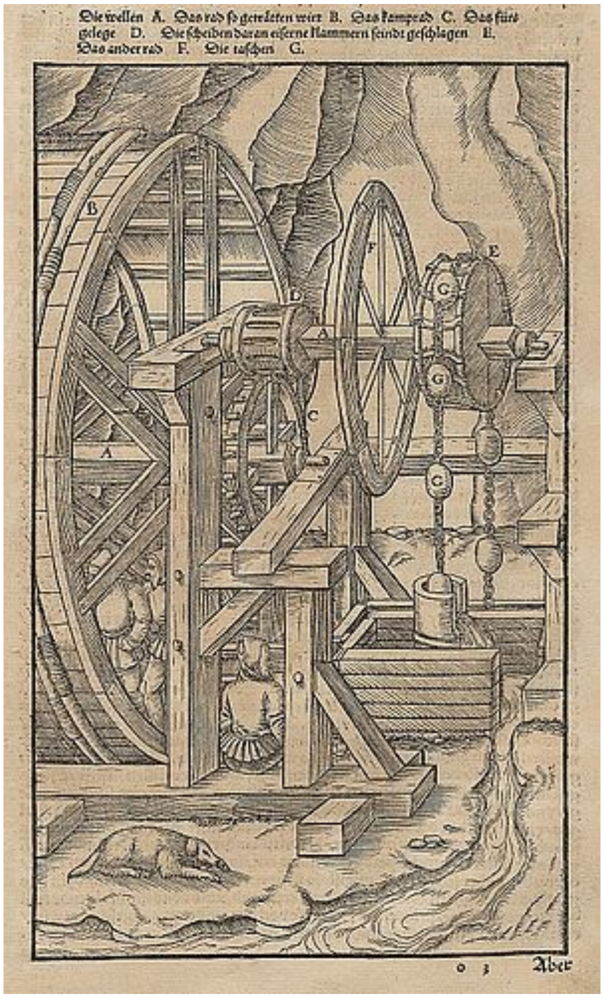
A **gear train** is a mechanical system formed by mounting gears on a frame so the teeth of the gears engage.

Gear teeth are designed to ensure the pitch circles of engaging gears roll on each other without slipping, providing a smooth transmission of rotation from one gear to the next.[1]

The transmission of rotation between contacting toothed wheels can be traced back to the Antikythera mechanism of Greece and the south-pointing chariot of China. Illustrations by the Renaissance scientist Georgius Agricola show gear trains with cylindrical teeth. The implementation of the involute tooth yielded a standard gear design that provides a constant speed ratio.

Features of gears and gear trains include:

- The ratio of the pitch circles of mating gears defines the speed ratio and the mechanical advantage of the gear set.
- A planetary gear train provides high gear reduction in a compact package.
- It is possible to design gear teeth for gears that are non-circular, yet still transmit torque smoothly.
- The speed ratios of chain and belt drives are computed in the same way as gear ratios. See bicycle gearing.




An Agricola illustration from 1580 showing a toothed wheel that engages a slotted cylinder to form a gear train that transmits power from a human-powered treadmill to mining pump.

## Mechanical advantage

Gear teeth are designed so the number of teeth on a gear is proportional to the radius of its pitch circle, and so the pitch circles of meshing gears roll on each other without slipping. The speed ratio for a pair of meshing gears can be computed from ratio of the radii of the pitch circles and the ratio of the number of teeth on each gear.

The velocity *v* of the point of contact on the pitch circles is the same on both gears, and is given by

$$v = r_A \omega_A = r_B \omega_B,$$

where input gear $A$ with radius $r_A$ and angular velocity $\omega_A$ meshes with output gear $B$ with radius $r_B$ and angular velocity $\omega_B$. Therefore,

$$\frac{\omega_A}{\omega_B} = \frac{r_B}{r_A} = \frac{N_B}{N_A}.$$

where $N_A$ is the number of teeth on the input gear and $N_B$ is the number of teeth on the output gear.

The **mechanical advantage** of a pair of meshing gears for which the input gear has $N_A$ teeth and the output gear has $N_B$ teeth is given by

$$\mathrm{MA} = \frac{N_B}{N_A}.$$

This shows that if the output gear $G_B$ has more teeth than the input gear $G_A$, then the gear train *amplifies* the input torque. And, if the output gear has fewer teeth than the input gear, then the gear train *reduces* the input torque.

If the output gear of a gear train rotates more slowly than the input gear, then the gear train is called a *speed reducer*. In this case, because the output gear must have more teeth than the input gear, the speed reducer amplifies the input torque.

## Analysis using virtual work

For this analysis, we consider a gear train that has one degree-of-freedom, which means the angular rotation of all the gears in the gear train are defined by the angle of the input gear.

The size of the gears and the sequence in which they engage define the ratio of the angular velocity $\omega_A$ of the input gear to the angular velocity $\omega_B$ of the output gear, known as the **speed ratio**, or **gear ratio**, of the gear train. Let $R$be the speed ratio, then

$$\frac{\omega_A}{\omega_B} = R.$$

The input torque $T_A$ acting on the input gear $G_A$ is transformed by the gear train into the output torque $T_B$ exerted by the output gear $G_B$. If we assume the gears are rigid and there are no losses in the engagement of the gear teeth, then the principle of virtual workcan be used to analyze the static equilibrium of the gear train.

Let the angle $\theta$ of the input gear be the generalized coordinate of the gear train, then the speed ratio $R$ of the gear train defines the angular velocity of the output gear in terms of the input gear:

$$\omega_A = \omega, \quad \omega_B = \omega / R.$$

The formula for the generalized force obtained from the principle of virtual work with applied torques yields:[2]

$$F_\theta = T_A \frac{\partial \omega_A}{\partial \omega} - T_B \frac{\partial \omega_B}{\partial \omega} = T_A - T_B / R = 0.$$

The *mechanical advantage* of the gear train is the ratio of the output torque $T_B$ to the input torque $T_A$, and the above equation yields:

$$\mathrm{MA} = \frac{T_B}{T_A} = R.$$

The speed ratio of a gear train also defines its mechanical advantage. This shows that if the input gear rotates faster than the output gear, then the gear train amplifies the input torque. And if the input gear rotates slower than the output gear, the gear train reduces the input torque.

彷彿清風掀開面紗，一睹『功率不變』之容顏◎

## Efficiency

Mechanical advantage that is computed using the assumption that no power is lost through deflection, friction and wear of a machine is the maximum performance that can be achieved. For this reason, it is often called the *ideal mechanical advantage* (IMA). In operation, deflection, friction and wear will reduce the mechanical advantage. The amount of this reduction from the ideal to the *actual mechanical advantage* (AMA) is defined by a factor called *efficiency*, a quantity which is determined by experimentation.

As an ideal example, using a block and tackle with six ropes and a 600 pound load, the operator would be required to pull the rope six feet and exert 100 pounds of force to lift the load one foot. Both the ratios $F_{out}$ / $F_{in}$ and $V_{in}$ / $V_{out}$ from below show that the IMA is six. For the first ratio, 100 pounds of force in results in 600 pounds of force out; in the real world, the force out would be less than 600 pounds. The second ratio also yields a MA of 6 in the ideal case but fails in real world calculations; it does not properly account for energy losses. Subtracting those losses from the IMA or using the first ratio yields the AMA. The ratio of AMA to IMA is the mechanical efficiency of the system.

## Ideal mechanical advantage

The *ideal mechanical advantage* (IMA), or *theoretical mechanical advantage*, is the mechanical advantage of a device with the assumption that its components do not flex, there is no friction, and there is no wear. It is calculated using the physical dimensions of the device and defines the maximum performance the device can achieve.

The assumptions of an ideal machine are equivalent to the requirement that the machine does not store or dissipate energy; the power into the machine thus equals the power out. Therefore, the power P is constant through the machine and force times velocity into the machine equals the force times velocity out–that is,

$$P = F_{in} v_{in} = F_{out} v_{out}.$$

The ideal mechanical advantage is the ratio of the force out of the machine (load) to the force into the machine (effort), or

$$IMA = \frac{F_{out}}{F_{in}}.$$

Applying the constant power relationship yields a formula for this ideal mechanical advantage in terms of the speed ratio:

$$IMA = \frac{F_{out}}{F_{in}} = \frac{v_{in}}{v_{out}}.$$

The speed ratio of a machine can be calculated from its physical dimensions. The assumption of constant power thus allows use of the speed ratio to determine the maximum value for the mechanical advantage.

## Actual mechanical advantage

The *actual mechanical advantage* (AMA) is the mechanical advantage determined by physical measurement of the input and output forces. Actual mechanical advantage takes into account energy loss due to deflection, friction, and wear.

The AMA of a machine is calculated as the ratio of the measured force output to the measured force input,

$$AMA = \frac{F_{out}}{F_{in}},$$

where the input and output forces are determined experimentally.

The ratio of the experimentally determined mechanical advantage to the ideal mechanical advantage is the efficiency η of the machine,

$$\eta = \frac{AMA}{IMA}.$$