樹莓派, 樹莓派之學習, 樹莓派之教育

時間序列：生成函數《八》

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假使有人議論著『〇』性質，說它『全方位』！講它『運行周天』！讚它『四時都不二』………！那麼有人『問』也，它『存在嗎』？它『能證明嗎』？它『唯此形式嗎』………？此所以『存在性定理』

Existence theorem

In mathematics, an **existence theorem** is a theorem with a statement beginning 'there exist(s) ..', or more generally 'for all *x*, *y*, … there exist(s) …'. That is, in more formal terms of symbolic logic, it is a theorem with a prenex normal form involving the existential quantifier. Many such theorems will not do so explicitly, as usually stated in standard mathematical language. For example, the statement that the sine function is continuous; or any theorem written in big O notation. The quantification can be found in the definitions of the concepts used.

A controversy that goes back to the early twentieth century concerns the issue of purely theoretic existence theorems, i.e., theorems depending on non-constructive foundational material such as the axiom of infinity, the axiom of choice, or the law of excluded middle. Such theorems provide no indication as to how to exhibit, or construct, the object whose existence is claimed. From a constructivist viewpoint, by admitting them mathematics loses its concrete applicability.[1] The opposing viewpoint is that abstract methods are far-reaching, in a way that numerical analysis cannot be.

'Pure' existence results

An existence theorem is purely theoretical if the proof given of it doesn't also indicate a construction of whatever kind of object the existence of which is asserted. Such a proof is non-constructive, and the point is that the whole approach may not lend itself to construction.[2] In terms of algorithms, purely theoretical existence theorems bypass all algorithms for finding what is asserted to exist. They contrast with “constructive” existence theorems.[3] Many constructivist mathematicians work in extended logics (such as intuitionistic logic) where such existence statements are intrinsically weaker than their constructive counterparts.

Such purely theoretical existence results are in any case ubiquitous in contemporary mathematics. For example, John Nash‘s original proof of the existence of a Nash equilibrium, from 1951, was such an existence theorem. In 1962 a constructive approach was found.[4]

Constructivist ideas

From the other direction there has been considerable clarification of what constructive mathematics is; without the emergence of a ‘master theory’. For example, according to Errett Bishop‘s definitions, the continuity of a function such as sin *x* should be proved as a constructive bound on the modulus of continuity, meaning that the existential content of the assertion of continuity is a promise that can always be kept. One could get another explanation from type theory, in which a proof of an existential statement can come only from a *term* (which we can see as the computational content).

之淵源流長也。

算數基本定理

算術基本定理，又稱為**正整數的唯一分解定理**，即：每個大於1的自然數均可寫為質數的積，而且這些素因子按大小排列之後，寫法僅有一種方式。例如: $6936 = 2^3 \times 3 \times 17^2$，$1200 = 2^4 \times 3 \times 5^2$。

算術基本定理的內容由兩部分構成：

- 分解的存在性：
- 分解的唯一性，即若不考慮排列的順序，正整數分解為素數乘積的方式是唯一的。

算術基本定理是初等數論中一個基本的定理，也是許多其他定理的邏輯支撐點和出發點。

誠如『建構』者言，且從『□』觀『四面』起始！『去四角』可得『八方』！『潤其棱』已有『十六邊』！……… 如是為之，『○』豈不可至乎！！？？『解構』者存疑焉？？！！『□能是○』乎？『幾時□將變○』乎？『已變○之□存在』乎？

因是假設一個『隨機變數』 X 有兩個『機率母函數』 $G_X^{\square}(z), \ G_X^{\circ}(z)$ ，而且 $G_X^{\square}(z) = G_X^{\circ}(z)$ ，那麼依據『泰勒定律』

Taylor's theorem

In calculus, **Taylor's theorem** gives an approximation of a *k*-times differentiable function around a given point by a *k*-th order **Taylor polynomial**. For analytic functions the Taylor polynomials at a given point are finite order truncations of its Taylor series, which completely determines the function in some neighborhood of the point. The exact content of "Taylor's theorem" is not universally agreed upon. Indeed, there are several versions of it applicable in different situations, and some of them contain explicit estimates on the approximation error of the function by its Taylor polynomial.

Taylor's theorem is named after the mathematician Brook Taylor, who stated a version of it in 1712. Yet an explicit expression of the error was not provided until much later on by Joseph-Louis Lagrange. An earlier version of the result was already mentioned in 1671 by James Gregory.[1]

Taylor's theorem is taught in introductory level calculus courses and it is one of the central elementary tools in mathematical analysis. Within pure mathematics it is the starting point of more advanced asymptotic analysis, and it is commonly used in more applied fields of numerics as well as in mathematical physics. Taylor's theorem also generalizes to multivariate and vector valued functions $\boldsymbol{f}\colon \mathbb{R}^n \to \mathbb{R}^m$ on any dimensions *n* and *m*. This generalization of Taylor's theorem is the basis for the definition of so-called jets which appear in differential geometry and partial differential equations.

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Taylor's theorem in one real variable

Statement of the theorem

The precise statement of the most basic version of Taylor's theorem is as follows:

Taylor's theorem.[2][3][4] *Let* k $\geq$ *1 be an integer and let the function* f *:* ***R*** $\to$ ***R*** *be* k *times differentiable at the point* a $\in$ ***R****. Then there exists a function* h_k *:* ***R*** $\to$ ***R*** *such that*

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(k)}(a)}{k!}(x-a)^k + h_k(x)(x-a)^k,$$

$$\text{and} \quad \lim_{x \to a} h_k(x) = 0.$$ *This is called the* ***Peano form of the remainder****.*

The polynomial appearing in Taylor's theorem is the ***k*-th order Taylor polynomial**

$$P_k(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(k)}(a)}{k!}(x-a)^k$$

of the function *f* at the point *a*. The Taylor polynomial is the unique "asymptotic best fit" polynomial in the sense that if there exists a function h_k : **R** → **R** and a *k*-th order polynomial *p* such that

$$f(x) = p(x) + h_k(x)(x-a)^k, \quad \lim_{x \to a} h_k(x) = 0,$$

then $p = P_k$. Taylor's theorem describes the asymptotic behavior of the **remainder term**

$$R_k(x) = f(x) - P_k(x),$$

which is the approximation error when approximating *f* with its Taylor polynomial. Using the little-o notation the statement in Taylor's theorem reads as

$$R_k(x) = o(|x-a|^k), \quad x \to a.$$

………

Relationship to analyticity

Taylor expansions of real analytic functions

Let $I \subset \mathbf{R}$ be an open interval. By definition, a function $f : I \to \mathbf{R}$ is real analytic if it is locally defined by a convergent power series. This means that for every $a \in I$ there exists some $r > 0$ and a sequence of coefficients $c_k \in \mathbf{R}$ such that $(a - r, a + r) \subset I$ and

$$f(x) = \sum_{k=0}^{\infty} c_k (x-a)^k = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots, \qquad |x-a| < r.$$

In general, the radius of convergence of a power series can be computed from the Cauchy–Hadamard formula

$$\frac{1}{R} = \limsup_{k\to\infty} |c_k|^{\frac{1}{k}}.$$

This result is based on comparison with a geometric series, and the same method shows that if the power series based on a converges for some $b \in \mathbf{R}$, it must converge uniformly on the closed interval $[a - r_b, a + r_b]$, where $r_b = |b - a|$. Here only the convergence of the power series is considered, and it might well be that $(a - R, a + R)$ extends beyond the domain I of f.

The Taylor polynomials of the real analytic function f at a are simply the finite truncations

$$P_k(x) = \sum_{j=0}^{k} c_j (x-a)^j, \qquad c_j = \frac{f^{(j)}(a)}{j!}$$

of its locally defining power series, and the corresponding remainder terms are locally given by the analytic functions

$$R_k(x) = \sum_{j=k+1}^{\infty} c_j (x-a)^j = (x-a)^k h_k(x), \qquad |x-a| < r.$$

Here the functions

$$\begin{cases} h_k : (a-r, a+r) \to \mathbf{R} \\ h_k(x) = (x-a) \sum_{j=0}^{\infty} c_{k+1+j} (x-a)^j \end{cases}$$

are also analytic, since their defining power series have the same radius of convergence as the original series. Assuming that $[a - r, a + r] \subset I$ and $r < R$, all these series converge uniformly on $(a - r, a + r)$. Naturally, in the case of analytic functions one can estimate the remainder term $R_k(x)$ by the tail of the sequence of the derivatives $f'(a)$ at the center of the expansion, but using complex analysis also another possibility arises, which is described below.

果然隨機變數 X 具機率分布的『存在』與『唯一』性也耶◎